Beam Propagation Analysis of the Nonlinear Tapered Optical Waveguide

Hwei-Yuan Liu and Way-Seen Wang

***Abstract*—A study of the transient behavior of nonlinear tapered optical waveguide is presented. The Fresnel equation with an input Gaussian field distribution is solved numerically using a combination of the semivectorial-polarized finite difference method and the Runge-Kutta method. The calculated results show that the tapered nonlinear waveguides have better waveguiding characteristics in potential applications.**

I. Introduction

In the past decade, the properties of nonlinear guided waves supported by nonlinear media have been studied extensively [1]–[4]. Intensity-dependent field pattern can be used for waveguide thresholding and waveguide switching operation, which lead to devices such as optical limiters and light-controlled spatial scanner to be realizable [2]. Hence, it is widely recognized that optical, especially all-optical, waveguide devices may become important components in communication and signal processing systems.

Previous works [1]–[4] on the propagation of nonlinear wave appears interesting in the case of a waveguide bounded by nonlinear media for spatial soliton emission and some novel effects are observed. Recently, it was reported that with a linear taper in the input waveguide [5], the optical transmission efficiency is increased. However, the wave propagation behavior of nonlinear tapered waveguides, especially in the early stage of nonlinear evolution, to our knowledge, have not be studied in detail. In this work, the beam propagation method with a semivectorial approximation [6] is used to analyze the "transient" behavior of a nonlinear tapered waveguide.

The numerical methods commonly used for solving the Fresnel equation, derived from the Helmholtz equation, are the fast Fourier transform method (FFTM) and the finite difference method (FDM). However, it was reported that the equation discretized by FDM is more efficient and stable than that by FFTM [7]. Usually, FDM starts with a discretization of the Fresnel equation, including both the transverse and the propagation components, by the central difference scheme [7], [8]. However, the Crank-Nicolson scheme for the propagation component is implicit, which takes a lot of computing time, especially when the full-wave analysis is considered. Recently, to save the computing time, the implicit Crank-Nicolson was replaced by the explicit Runge-Kutta fourth order formulas [8].

Manuscript received August. 29, 1994. This work was supported by National Science Council, Taipei, Taiwan, Republic of China under Contract No. NSC83-0417-E-002-010.

The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei 10617, Taiwan, ROC.

IEEE Log Number 9407745.

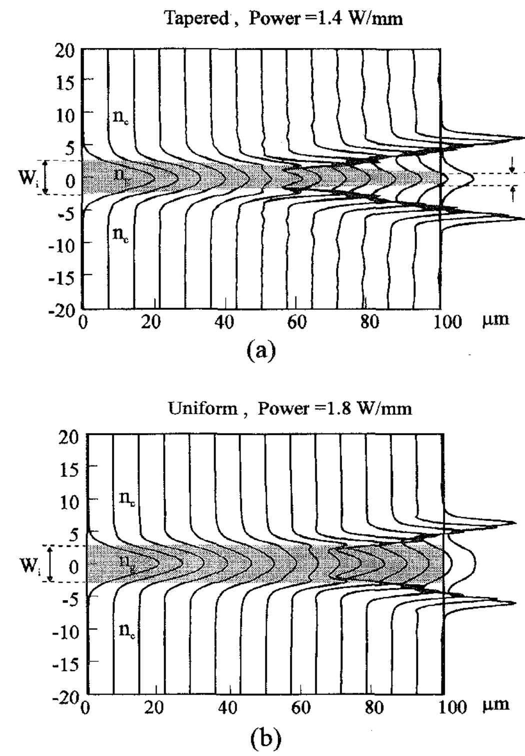

Fig. 1. The typical field distributions of (a) tapered and (b) uniform nonlinear waveguides.

Previous demonstration [5] was for a linear partial differential equation. However, it is well known that the Runge-Kutta method is derived for ordinary differential equations irregardless of whether the equation is linear or nonlinear. In this work, it is shown that the explicit method [5] is also good for the nonlinear partial differential equations, especially that for the nonlinear tapered waveguide mentioned previously.

II. Simulation and Results

Consider an optical waveguide of refractive index n_g bounded by two nonlinear media of indices $n_c = n_0 + \alpha|E|^2$ as shown in Fig. 1, where α is the nonlinear coefficient and E is the propagating field. With the assumption that the guided optical wave is slowly varying and paraxially propagating, the Helmholtz equation can be reduced to the Fresnel equation as given by

$$2jk_0n_0\frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k_0^2[n^2(x,y,z) - n_0^2]E \quad (1)$$

1051–8207/95$04.00 © 1995 IEEE

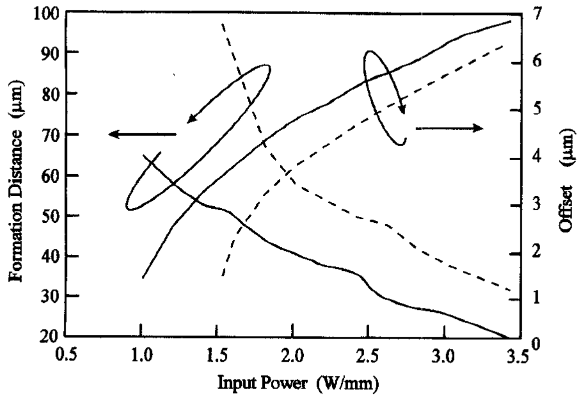

Fig. 2. The formation distance (left) and the offset (right) of solitons. Solid line: tapered waveguide; dashed line: uniform waveguide.

where k_0 is the free space wavelength number. The Fresnel equation is discretized as a system of ordinary differential equations. The stability and accuracy of our program were tested using some typical values of the previous experiment [5].

For simplicity, we consider a two-dimensional tapered nonlinear waveguide of widths W_i and W_o at the input and output ends, respectively and a uniform nonlinear waveguide of width W_i. Assume the waveguide is initially excited with a laser light of wavelength $\lambda = 1.3$ μm, power $P_i = 0.4 \sim 3.2$ W/mm, and Gaussian field distribution [4] as given by

$$E(x, z = 0) = E_0 \exp\left(-\frac{4x^2}{W_i^2}\right) \tag{2}$$

For the numerical calculation, we use $W_i = 5$ μm, $W_0 = 1$ μm, $n_0 = 1.55, n_g = 1.57, \alpha = 3.3776 \times 10^{-12}$ m^2/V^2 [9], [10], $\Delta x = \Delta z = 0.2$ μm, and the waveguide length L is 100 μm. From our calculated results, it is found that within the power of interest, i.e. the power large enough to have one or two solitons emitted, our explicit scheme is applicable.

Fig. 1 shows the typical field distributions of the tapered and the uniform nonlinear waveguides. As can be seen from the figure, the distance required for the formation of the solitons from the initial Gaussian excitation is shorter for a tapered waveguide even when launched with a lower input power. Fig. 2 shows the formation distance of the solitons vs. the input power. When the input power is low, no solitons are emitted. Only when the input power is greater than a certain threshold will the first solitons be emitted. Obviously, the tapered waveguide has a lower threshold than that of the uniform one. This is converging of the optical energy in a tapered waveguide increases the nonlinearity near the waveguide interfaces, and therefore, enhances the formation of the solitons. As the input power increases further, the formation distance decays very rapidly, which is due to the fact that the input energy is just about equal to that of the first solitons. For an even larger power, the formation distance decays less rapidly but with an oscillatory behavior. This due to the fact that the excess energy included in the initial Gaussian excitation interacts with the emitted solitons, which drags slightly the emission of the solitons.

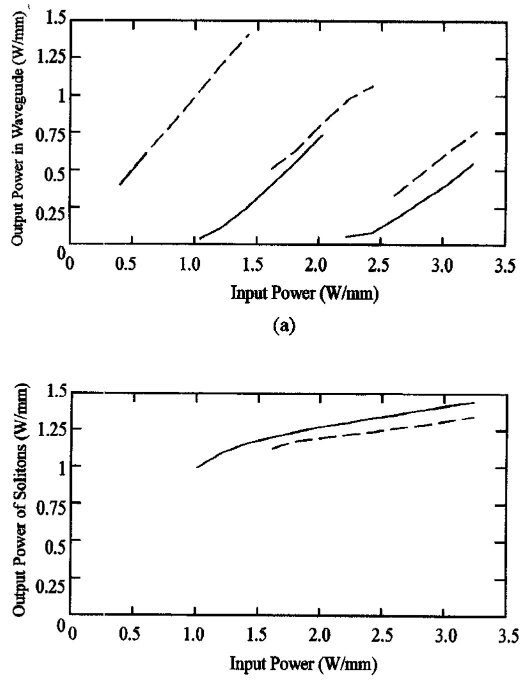

Fig. 3. (a) The output power trapped in waveguide and (b) the output power of emitted solitons. Solid line: tapered waveguide; dashed line: uniform waveguide.

Fig. 2 also shows the distance of the emitted soliton offset from the central axis of the waveguide. When the input power is lower than the threshold, the offset is zero, just as expected. As the power increases, it can be seen that the tapered waveguide has a larger offset than that of the uniform one. And the larger the input power, the more the offset, which means a tapered waveguide can greatly enhance the spatial angle when it is used in a power-controlled optical scanner.

Fig. 3 shows the power trapped inside the waveguide (P_g) and the power emitted by the solitons (P_s) for the tapered and the uniform waveguides. From Fig. 3(a), the sequence of threshold is clearly seen [1], also the first threshold occurs for the first emission of solitons, but the tapered waveguide emits solitons earlier and also switches off clearer than the uniform waveguide does. From this figure, following up to the second threshold, the corresponding second solitons are also emitted. Fig. 3(b) shows that the tapered waveguide has a lower threshold for soliton emission and a higher efficiency of soliton transmission. For example, when $P_i = 1.0$ W/mm, the first solitons are emitted from the tapered waveguide and the transmission efficiency $\eta_s (= P_s/P_i)$ is 99.07%, however when $P_i = 1.6$ W/mm, the first solitons are emitted from the uniform waveguide and its η_s is only 78.95%. Moreover, when $P_i ==$ 2.2 and 2.6 W/mm, there are second solitons emitted from the tapered and the uniform waveguides, respectively.

III. Conclusion

In conclusion, the transient behavior of a nonlinear tapered optical waveguide with an input Gaussian field distribution is presented. The calculated results show that the nonlinear tapered waveguide has a shorter soliton formation distance, a larger soliton offset. Varying the input power continuously, the device can be used as a power-controlled scanner. The threshold behavior can be used as logic elements for all-optical switching circuits [1]. Moreover, the combination of the Runge-Kutta and the semivectorial-polarized finite difference method is a simple and effective way for nonlinear beam propagation analysis. Further application on the beam propagation analysis of other waveguide devices is of great interest in the future.

REFERENCES

[1] E. M. Wright, G. I. Stegeman, and C. T. Seaton, "Multisoliton emission from a nonlinear waveguide," *Phys. Rev. A*, vol. 34, no. 5, pp. 4442–4444, 1986.
[2] J. V. Moloney, J. Ariyasu, C. T. Seaton, and G. I. Stegeman, "Stability of nonlinear stationary waves guided by a thin film bounded by nonlinear media," *Appl. Phys. Lett*, vol. 48, no 13, pp. 826–828, 1986.
[3] P. Varatharajah, J. V. Moloney, A C Newell, and E. M. Wright, "Stationary nonlinear waves guided by the films bounded by nonlinear diffusive media," *J. Opt. Soc. Am. B.*, vol. 10, no. 1, pp. 46–54, 1993.
[4] N. N. Akhmediev, V. I. Korneev, and U. V. KuZ'menko, "Excitation of nonlinear surface waves by Gaussian light beams," *Sov. Phys. JETP*, vol. 61, no. 1, pp. 62–67, 1985.
[5] P. K. Wei, H. Y. Liu, and W. S. Wang, "Beam propagation analysis of a tapered proton-exchanged lithium niobate optical waveguide," *IEEE Microwave and Guided Wave Lett.*, vol. 4, no. 2, pp. 40–42, 1994.
[6] P. L. Liu and B J. Li, " Semivectorial beam-propagation method for analyzing polarized modes of rib waveguides," *IEEE J Quantum Electron.*, vol. QE-28, no. 4, pp. 778–782, 1992.
[7] R. Scarmozzino and R M. Osgood, Jr., "Comparison of finite-difference and Fourier-transform solutions of the parabolic wave equation with emphasize on integrated optics application," *J. Opt Soc Amer A*, vol. 8, no. 5, pp. 724–731, 1991.
[8] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes The Art of Scientific Computing*. London: Cambridge University, pp. 550–554.
[9] C T. Seaton, J. D. Valera, R. L. Shoemaker, G. I. Stegeman, J. T. Chilwell, and S. D Smith, "Calculations of nonlinear TE waves guided by thin dielectric films bounded by nonlinear media," *IEEE J. Quantum Electron.*, vol. QE-21, pp. 774–783, 1985.
[10] T. T. Shi and S. Chi, "Nonlinear Y-junction coupler," *Electron. Lett.*, vol. 26, no. 5, pp. 339–340, 1990.